STUDY AND DISCUSSION OF MILITARY GRIDS

by

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1. Purpose of Study

The purpose of the following study is to determine the characteristics required in a military grid and to select a system most nearly answering these requirements. Marked disadvantages are inherent in most grid systems now used. These disadvantages are complicated by the existence of many incompatible systems.

2. Existing Conditions

2.01. The polyconic military grid is prescribed by Section VII, AR 300-15, for use on all military maps of the United States. This system is laid out in zones 9 degrees wide in longitude with 1 degree of overlap between zones. It is so inaccurate at long ranges in certain directions that it cannot be used satisfactorily for the control of the fire of coast artillery weapons or heavy field artillery.

2.02. Because of this inaccuracy, the coast artillery harbor defense grid for areas in the neighborhood of the harbor defenses in the continental United States is also authorized in paragraph 28, Change No. 4, AR 300-15. This harbor defense grid system is a Lambert conic conformal designed particularly to serve the guns of the harbor defense concerned. It is not only not connected to the military grid system in the same area but is incompatible therewith.

2.03. Many other grid systems are in use not only in the United States but also in the rest of the world. Twenty states of the United States have

adopted the state plane coordinate systems measured in feet and especially designed to serve the particular state in question. Each such system is hardly extensible beyond the borders of the state without the introduction of material inaccuracies. The inclosed map shows the overall picture of the grid systems used in allied military operations during the recent campaigns. (Incl. 1)

2.04. It is obvious that the presence of so many systems complicates map preparation and imposes material confusing handicaps on actual combat operations. The presence of more than one grid system covering one area presents no particular problem in peace time or on maneuvers involving but one arm. When the fog of war confuses men's minds, the presence of several coordinate systems in one area for use of different arms is fraught with potent opportunity for disasters resulting from uncoordinations attributed to mistakes in using the military grid. Involved in this matter are branch pride and branch stubbornness, each branch feeling justified in having a special grid system designed to the particular capabilities and needs of that branch. An example of this occurred in the United Kingdom where the British coast artillery, Navy and Air Force covered the coastal area with three incompatible, incommensurable grid systems. Intolerable confusion which resulted from the use of these grids during the numerous German air raids in the Battle of Britain makes it highly probable that these conflicting systems would have led to at least a few local disasters had there been an invasion of Great Britain .

3. Basic Requirements

3.01. Primary Purpose of a Grid. The primary purpose served by the military grid on a map is to provide quick solutions to problems of distance and azimuth for the firing of weapons. It provides a quick simple means for referring to spot locations and for designating targets. It is an essential tool in coordination of military operations.

3.02. The coordination of the efforts of the many arms used on land, sea, and air, is a problem so complex as to make mandatory a single simple solution for problems of target designation and determination of range and azimuth. This requirement is believed to be so important in war that the use of a single system of limited but adequate accuracy is held to be better than the simultaneous use of two incompatible but otherwise more accurate systems.

3.03. The characteristics of the using arms and weapons which affect the design of the system to be adopted involve relatively little research. As a general rule, it has been assumed that permanently emplaced batteries will be more accurate in their fire than batteries temporarily emplaced in the fields. Therefore, the following table appears to provide sufficient criteria to determine the desirable accuracies of the grid system adopted.

Probable Errors of Different Caliber
Permanently Emplaced Guns at Ranges Shown

Range in Yards	Probable Errors in Yards						Minimum Probable Relative Errors	
	6" Gun		8" Gun		16" Gun			
	Range	Defl.	Range	Defl.	Range	Defl.	Range	Deflection
10,000	22	2	68	3	18	3	1:555	1:5,000
15,000	35	4	70	5	28	4	1:535	1:3,750
20,000	52	6	73	8	40	6	1:500	1:3,333
25,000	68	8	77	13	52	7	1:480	1:3,571
30,000			83	19	63	9	1:476	1:3,333
35,000					73	10	1:479	1:3,500
40,000					80	10	1:500	1:4,000
45,000					*77	*7	*1:584	*1:6,428

* These values appear unusual.

4. Desirable Characteristics

4.01. Primarily a grid system should be accurate enough for all weapons and all military uses other than for very long distance missiles, should be quickly applicable to any previously ungridded native,map, should yield readily to simple computing methods and should provide simple numerical designators for location of targets.

4.02. Plane System. The system of coordinates desired is one with which all computations for the most accurate artillery firing can be simply yet accurately performed and especially one in which the integrity of angles is preserved. A mathematically exact graticule, such as that presented by the meridians and parallels, requires the use of geodetic functions to solve the spherical triangles involves, and entails a long, time-consuming complicated computation. Moreover, due to the convergence of the meridians, the arc of the parallel intercepted between any two meridians becomes shorter as the latitude increases. Complicated geodetic formulas would be necessary in the computation of any distance except one along a meridian. Complex fire control instruments would be needed, manned by personnel highly trained in a branch of advanced mathematics. Neither the personnel, the instruments, nor the time are normally available. As a consequence, the system adopted should be one in which plane trigonometry can be employed in the solution of triangles. In such a plane system for general application to large areas, the simplest and quickest computations can be secured through use of a grid network of equally spaced parallel and mutually perpendicular lines.

4.03. Grid Accuracy. A high degree of accuracy is, of course, desirable. However, grid accuracies which are greatly in excess of the accuracies for the most precise weapon using the grid appear to be neither necessary nor practicable. By reference to paragraph 3.03 above, it will be noted that the probable errors of artillery weapons are much greater in range than are their probable errors in deflection. The minimum probable error of permanently emplaced guns rarely is less than 1/555 in range and 1/5,000 in deflection. Consequently, a suitable military grid should be one designed to conform to these minimum probable errors.

4.04. Adaptability to Various Projections. The grid system selected should be adaptable for use on native maps without complicated recomputation or redrafting of that map. There are many map projections used in the making

of large scale maps throughout the world. It is desirable to be able to overprint the adopted grid system on any or all of these projections without the introduction of errors in range and azimuth beyond that probable in good artillery practice.

4.05. Unit of Measure. Three general systems of linear measure are commonly encountered on maps and in grids: the metric system, the so-called English system, and the nautical system. Mixtures of these systems unfortunately are prevalent. This matter is further complicated by the fact that three differing elements are involved - map quantities, grid quantities, and the quantities employed by the using arms and weapons.

a. Map quantities include azimuths, horizontal distances, contour intervals, and underwater depths. To the map user, the unit of measure in which horizontal distances on a map are expressed is not particularly important, as the conversion from map distances to ground distances is frequently done graphically against either an appropriately graduated bar scale on the map or range scale. However, the unit of linear measure used in the basic survey of the map may complicate the computation and compilation of trig lists for fire control. This latter operation is already quite complex due to the differing origins of longitude, datum planes, spheroids and schemes of projection, and other variations encountered in the native surveys of the world. Thus, the conversion from one unit of linear measure to another incommensurable unit adds an operation subject to mistakes and affecting final accuracies.

b. Contour intervals and spot elevations should, but may not, be in the same unit of measure as the horizontal distances, in order to provide for the ready calculation of true slant ranges, defilade, mask, profile, etc.

c. Underwater depths are generally expressed either in meters or in fathoms, although shoal water depths may be also expressed in feet. It is highly desirable that these units of measure be the same as the hori-

profiles and beach gradients.

d. The military grid, while essentially concerned with angles and horizontal distances, must be precisely related to the computed geographic positions. The necessary correlation between the vertical unit of measure and the horizontal unit of measure on the grid as indicated in b above, is essential for the quick solution of problems involving defilade, mask and true gun target distance.

e. The using arms and weapons are not entirely coordinated in the units of measure employed in the laying of the piece. The Coast Artillery measures azimuths in degrees and hundredths of a degree from grid south as the origin. The Field Artillery measures angles in mils, with field orientation off base circles. The Coast Artillery measures its range in yards, while the Field Artillery may measure it in yards or meters. Due to the tangent relationship of the mil, Field Artillery can readily transpose from angular measure to linear distance in either meters or yards. Each artillery weapon is served and laid by employing a multiplicity of tabular information, plotting tools, and gunner's instruments. All these things must be related to the unit of measure selected for map and grid quantities. At the present moment, due to the recent tremendous concentration of field artillery weapons in the European campaigns and to the use of the metric system throughout in that area, our Field Artillery is well equipped and trained in the use of the metric system. The Seacoast Artillery of the United States, including the Panama Canal and Oahu, are not so equipped or trained. They use the yard-hundredths of a degree system, except in the coast defenses of San Diego where the coast Defense grid is graduated in feet rather than in yards.

f. Existing Conditions. The majority of the large scale maps of the world are made on the metric system. Exceptions to this rule are the

United States, Canada, Australia, United Kingdom, Union of South Africa, India, Melanesia, and Middle East. The following table shows the unit of measure of native maps and military grids employed in operational areas of the recent campaigns.

	MAP UNITS			GRID UNITS	
AREA	HORIZONTAL UNIT (Map Bar Scale)	VERTICAL UNIT (Map Contour)	DEPTH UNIT (Bathymetric Contour)	GRID UNIT	GRID TYPE
1.France	Meter	Meter	Meter	Meter	Lambert
2. Germany	Meter	Meter	Meter	Meter	Trans.Merc.
3.Italy	Meter	Meter	Meter		
4.Tunisia	Meter	Meter	Meter		
5.Libya	Meter	Meter	Meter	Meter	Trans.Merc.
6.Egypt	Meter	Meter	Fathom	Meter	Trans.Merc.
7.Okinawa	Meter, Cho	Meter	Meter	Meter	Unknown
8.Burma	Miles	Foot	Fathom	Yard	Lambert
9.China	Li, Meter	Meter	Foot	Meter	Lambert
10.Russia	Meter	Meter	Foot	Meter	Trans.Merc.
11.Hawaii	Mile	Foot	Fathom	Yard	Polyconic
12.Philippines	Mile	Foot	Fathom	Yard	Polyconic
13.Poland	Meter	Meter	?	Meter	Stereographic
14.Holland	Meter	Meter	Meter	Meter	Stereographic
15.Belgium	Meter	Meter	Meter	Meter	Bonne
16.Japan	Meter, Cho	Meter	Meter	Meter	Trans.Merc.

4.06. Width of Zone. An inherent fault of any system of plane coordinates applied to the surface of a spheroid is the fact that inaccuracies increase as the zone is extended east and west or in other cases, as the belt is extended north and south. It is also desirable, although not entirely necessary, to keep at a minimum the deviation of grid north from true north. This deviation likewise increases materially as the distance from the central meridian increases. These inaccuracies can be kept within reasonable limits by the adoption of narrow grid zones. It has been stated only semi-facetiously that there are three military engineering axioms:

a. It always rains in war.

b. It's always too cloudy to get aerial mapping photographs.

c. Battles are always fought on grid junctions.

This last axiom is spoken from the depths of bitter experience, rendering it obvious that a grid zone should include as much area as possible so as to obviate too frequent junctures between grid zones on the battlefield. However, this desirable criterion cannot be widely applied without including intolerable inaccuracies of the grid. Elimination of the undesirable features consequent upon fighting a battle on a grid juncture can be partially accomplished by providing for overlap between grid zones. The 9 degree width of the military grid system presently prescribed for use in the United States introduces appreciable inaccuracies near the edges. A reduction in width to 6 degrees betters this situation materially. The computation of ranges and azimuths where the gun position is located in one grid zone and the target in another can be provided for by a half degree overlap between grid zones, enabling the coordinates of the gun zone to be extended a half a degree into the grid zone in which the target is located.

5. Comparison of Grids

5.01. Polyconic Grid. For military purposes, a grid may be regarded as a set of perfect squares ruled on a plane map, scale 1:1, and then transferred to the earth's surface. Evidently after being transferred to the earth's surface the squares will no longer be perfect; and distortions they will have received in being put on the surface of the earth will reflect the distortions of the projection used for the map.

a. The polyconic projection is defined as one which the central meridian and all parallels are mapped to scale and with true curvature. All other lines are stretched, the amount increasing as the square of the distance from the central meridian, and being greates t for north-south lines. Angular errors also appear, increasing with distance from the central meridian. Of course it is possible to compute these errors, at least roughly, and to allow

for them, and this is regularly done by engineer survey troops. But the corrections are generally considered beyond what is to be expected of artillery units in the field, and for that reason all mention of them is omitted in artillery technical manuals, even when survey procedures are discussed. It is proper, therefore, to compare the errors of the projection as shown in the following table, directly with the errors of the guns (Section 3.03).

Errors of U. S. Military Grid at 4° 30' from Central Meridian, and 30° Latitude

True Range (yds)	True Azimuth 0°				True Azimuth 45°				True Azimuth 90°			
	Range Error (yds)	Relative Error	Defl. Error (yds)	Relative Error	Range Error (yds)	Relative Error	Defl. Error (yds)	Relative Error	Range Error (yds)	Relative Error	Defl. Error (yds)	Relative Error
10,000	23	1:435	0	0	12	1:836	12	1:836	0	0	0	0
30,000	70	1:428	4	1:7500	35	1:859	35	1:859	0	0	0	0
40,000	93	1:430	8	1:5000	47	1:850	47	1:850	0	0	0	0

It is evident that at 40,000 yards, and an azimuth of 45°, the error of the grid in deflection is almost five times the probable error of a 16-inch gun. Errors of the above amount are characteristic of the so-called non-conformal projections, i.e., those in which the shapes, as well as the scale of small areas is distorted. Such non-conformal grids were widely used prior to World War I; but most of them have been abandoned in recent years, chiefly, no doubt, because of the breakdown of the old French Bonne grid during the War. In addition to its lack of conformality, the polyconic projection possesses the disadvantage that it has not been studied thoroughly from a mathematical standpoint. Hence the small corrections

needed for precise surveys are not known; the transformation from other grids to the polyconic is not known; not even the transformation from one polyconic belt to another has been studied.

b. So far as grid junctions are concerned, the polyconic is theoretically excellent; it can be extended indefinitely both north and south, so that the world can be divided up into meridianal strips. In practice, the present World Polyconic has two latitudinal junctions, one near 24°, due to failure to put the origin of the old U. S. grid sufficiently far south; the other is near 49°, and is due to inaccuracy in the old tables which amounts to 1.1 meters.

c. The polyconic grid is not well suited for foreign maps due to its lack of conformality. By the older laborious hand methods of grid plotting, this introduced no difficulties other than laborious calculations; but the use of the coordinatograph, for rapid plotting of grids and projections, requires a conformal grid.

5.02. Cassini-Soldner Grid. This is a non-conformal grid, very similar to the polyconic, and open to the same objections. It differs only in that the grid east-west lines, rather than the parallels are represented to true scale and with true curvature.

5.03. The Bonne Grid. Both meridians and parallels are represented to true scale on the Bonne projection; the error shows up on lines which run NE to SW, or NW to SE. It is just as bad as in the case of the other non-conformal projections; and this projection has been generally abandoned.

5.04. The Stereographic Grid. The stereographic grid may be briefly defined as a conformal grid (that is, one having zero angular distortions for small distances, and very small angular distortions for any distance) in which the scale error is zero on a standard circle. The fact that

this grid is conformal insures that within about 200 miles of the center-point, the error in deflection will be less than the error of the most accurate guns. The range error is considerably larger than the deflection error, but is considerably less than the range error of permanently emplaced artillery. Unfortunately, this grid cannot be extended more than 300 miles from its center point and grid zones are circular in shape. It is quite suitable for small roughly circular countries such as Poland, Holland, or Rumania, which use it; but on a world-wide basis it would lead to a great multiplicity of grid junctions, and points where three or more grids meet could not be avoided.

5.05. Lambert Grid. Like the stereographic, the Lambert is a conformal projection. It may be defined as a conformal projection in which the scale errors are zero along two parallels. It is well suited to the mapping of moderately large areas, and has been extensively used by the British and French, especially in recent years. The numerical values of the errors are similar to those for the Transverse Mercator given in the next paragraph. The errors are very small in deflection, as is necessary for an artillery grid. It is readily adaptable to gridding maps on other projections. In general a slight change of scale only would be required. On the other hand, the rapidly increasing grid declination makes it impracticable to extend the grid more than about 15° from the central meridian (except along the Equator, where grid declination is always zero). For this reason, the British and French were forced to introduce numerous zones, and to permit junctions of three grids at the same point. These numerous grid zones necessary, both north-south and east-west, make the Lambert undesirable for extensive world coverage.

5.06. Transverse Mercator Grid. The Transverse Mercator grid may be defined as a conformal grid in which the central meridian is represented by a straight line at true scale. It is well suited to large areas, and is being used by the Germans, Russians, British, and Japanese. The errors at a given point vary little in all azimuths and average values for different ranges are given in the following table for 30° north latitude and for $3\frac{1}{2}$° from the central meridian.

True Range (yds)	Transverse Mercator Grid Range Error (yds)	Relative Error	Transverse Mercator Grid Deflection Error (yds)	Relative Error
10,000	10	1:1000	0	0
30,000	30	1:1000	3	1:10000
40,000	40	1:1000	5.8	1:6900

Obviously, the grid is well suited to artillery purposes since the inevitable errors are thrown into range rather than deflection. The grid can be extended indefinitely in latitude like the polyconic. Hence it is never necessary to have a grid junction involving more than two grids (except, of course, near the Poles). Transformation of coordinates from one belt to another can be done by a formula already worked out. The formula is always the same, and is very simple in character. The grid declination will remain moderate throughout the belt. The grid can be readily adapted to use on other projections. Much theoretical work has already been done on this subject by a large group of mathematicians, including especially Professor W. K. Hristow. Extensive computations, especially for the Balkan countries, were done by the German High Command (O. K. H.) during World War II which could, in an emergency, be promptly utilized if the proposed projection is adopted. In addition,

much geodetic data of foreign areas on file at Army Map Service are on this system. The projection is well suited for converting data on various spheroids to a common basis. Traverse and lower orders of triangulation may be computed and adjusted directly on the grid due to its conformality. This feature, which is a large saving in field and office, is not practicable where a non-conformal projection such as the present Polyconic is used.

5.07. Tabular Comparison of Grids.

GRID SYSTEM	APPLICABILITY TO FOREIGN MAPS /	GRID JUNCTIONS	MAXIMUM RELATIVE RANGE ERRORS* (45,000 yds)	MAXIMUM RELATIVE DEFLECTION ERRORS* (45,000 yds)
Polyconic	Very poor	Few & simple	1/1228	1/2454
Cassini-Soldner	" "	" "	1/1228	1/2454
Bonne	" "	Many & complex	1/2454	1/615
Stereographic	Good	" "	1/2454	1/15,360
Lambert	"	" "	1/2416	1/7,663
Transverse Mercator	Excellent; much work already done	Few & simple	1/2416	1/7,663

/ A grid system is considered applicable to a foreign map if it can be put on most maps without changing map or grid except in scale.

* Range and deflection errors are maximum values within 160 miles from the center of the projection, whether the center is a line, as in the Transverse Mercator, Lambert, Cassini-Soldner, and Polyconic, or a point as with the other two. The figures are based on GSGS "Survey Computation". 160 miles is the approximate distance (at 40° of latitude) from the center of the proposed Transverse Mercator zones to the junctions, about 3° of longitude. The actual maximum errors of the present World Polyconic grid are considerably larger, since the grid zones are 9° in width.

6. Conclusions as to System to be Adopted.

6.01. The Lambert Orthomorphic projection is conformal but is not suitable as it requires grid zone junctures both north and south and east and west. The polyconic grid system now prescribed for use as military

grid on all maps of U. S. is inaccurate in both azimuth and distance. The greater inaccuracy is in azimuth and is more than the probable error in deflection of permanently emplaced guns. The transverse mercator grid is conformal and is immediately applicable without plottable error, to the majority of the map projections commonly encountered on the native maps of the world. The transverse mercator grid reduces inaccuracies to a point where they are compatible with the accuracies required by all modern artillery weapons. This grid is sufficiently accurate to eliminate the necessity for a special Coast Artillery grid in the vicinity of coast defense locations.

6.02. In view of the foregoing, a military grid system based on the transverse mercator projection applied to the local spheroid and measured in meters, or in the standard unit of the country concerned, should be applied in zones running from Latitude 80° N to Latitude 80° S, 6 degrees of longitude wide, with one degree of overlap ($\frac{1}{2}$ degree each side). The latitude of the origin is the equator. (Incl. #2). The false easting to be applied for each zone would be 500,000 meters or yards. Scale factor on the C. M. should be 0.9996. The zones should be numbered, commencing with Zone 1, with its western edge at 180° longitude, running east to 174° west longitude. Consecutively numbered zones continue eastward by successive steps of 6 degrees until reaching the point of beginning; these number designations being identical to the I.M.W. 1:1,000,000 layout. It will be noted that in certain countries where the native maps use the English units in elevations and contours, such as the U. S., Canada, Australia, India, the proposed grid system should be graduated in yards rather than meters. In certain training areas in the U. S., both metric and yard grids will be required for

training purposes. Where the metric grid is used in the domestic U.S., the spot elevations should be in meters and the contours should be converted to a metric interval, provided that such conversion of contours shall be limited to those maps to be used for metric training purposes.

7. Proposed Specifications

Projection:	Transverse Mercator.
Spheroid:	Same as that used to compute the triangulation of the area.
Unit:	Meter in most areas; yard in U. S. and other areas where the English system is firmly established.
Central Meridians:	3° E (or W) of Greenwich and every 6° thereafter.
Latitude of Origins:	0°
False Easting:	500,000 meters (or yards)
False Northing:	0 for northern hemisphere 10,000,000 for southern hemisphere.
Scale Factor:	0.9996
Zone Width:	6° of longitude (plus $\frac{1}{2}$° overlap at each edge)
Limits of Zones:	North: 80° latitude South: -80° latitude
Zone Numbering:	Commencing with Zone 1 east of 180° longitude, and continuing easterly around the earth. (Identical to I. M. W. system designation)
Limits of Tables:	North: 80° latitude South: -80° latitude

8. Implementing Actions and Costs

8.01. It should be noted that the application of the transverse mercator grid system should be progressive rather than instantaneous. Priorities for conversion are indicated as follows:

a. Military areas in the United States.

b. Mapping and map revision of foreign areas embraced in the 20-year strategic mapping plan approved by the War Department.

c. General areas of the United States as stocked by the Army Map Service.

d. Other maps of foreign areas as reissued.

Upon the issue of the new map in any area, the new grid will normally be shown in full, but to safeguard against the event of the occurrence of an emergency while a series is still in a state of partial conversion, the maps will carry marginal marks to permit easy plotting and overprinting of the old grid.

8.02. Since Army Map Service and map depots in the theaters now hold extensive stocks of maps carrying the expedient war-time grids, it is to be expected that the proposed standardization of the transverse mercator grid automatically renders obsolescent these stocks. The cost of a conversion in this respect is estimated as follows:

a. Cost of conversion of points and correction of drafting copy

(1)	United States areas	$ 99,500
(2)	European areas	$103,000
(3)	Other overseas areas	$ 22,000

b. Cost of replacement of stocks to be retired

(1)	United States areas	$ -0-
(2)	Overseas areas	$ 10,000

Since all United States maps are to be converted to military scales (1/25,000, 1/50,000), new stocks are to be prepared in any event. Overseas accumulation of war time maps, it has been reported, are being salvaged except for small reserves which will probably not be replaced. The

$10,000 figure should be ample to provide for all requisitions directly attributable to the change in grid. The cost of now authorizing the proposed conversion is properly to be weighed against the much greater cost that would be borne should circumstances require the conversion ten years hence. The conversion must ultimately be made in view of the inadequacies of the present medley of grids to suffice for the anticipated requirements of another war. It is considered unquestionable that the cost of the conversion should be accepted now if it be agreed by the General Staff that the transverse mercator grid is in fact the correct design for the future, based on what can now be discerned as to future characteristic requirements.

8.03. It is suggested that the views of the Navy Department be obtained prior to final standardization in view of the application of the military grid to maps and charts for amphibious operations. It would also be desirable to coordinate the design so far as possible with the Director of Military Survey, War Office, London, who has expressed his general views in an informal letter (Incl. #3).